

Multipartite entanglement of fermionic systems in noninertial frames

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Abstract

The bipartite and tripartite entanglement of a 3-qubit fermionic system when one or two subsystems accelerated are investigated. It is shown that all the one-tangles decrease as the acceleration increases. However, unlike the scalar case, here one-tangles $\mathcal{N}_{C_I(AB_I)}$ and $\mathcal{N}_{C_I(AB)}$ never reduce to zero for any acceleration. It is found that the system has only tripartite entanglement when either one or two subsystems accelerated, which means that the acceleration doesn't generate bipartite entanglement and doesn't effect the entanglement structure of the quantum states in this system. It is of interest to note that the π -angle of the two-observers-accelerated case decreases much quicker than that of the one-observer-accelerated case and it reduces to a non-zero minimum in the infinite acceleration limit. Thus we argue that the qutrit systems are better than qubit systems to perform quantum information processing tasks in noninertial systems.

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I. INTRODUCTION

Quantum entanglement is both the central concept and the most desirable resources for a variety of quantum information processing tasks [1–3], such as quantum teleportation, super-dense coding, entanglement-based quantum cryptography, error correcting codes, and quantum computation. In the last decade, although many efforts have been made on the study of the properties of entanglement, the good understanding of such a resource is only limited in bipartite systems. There's no doubt that the multipartite entanglement is a valuable physical resource in large scale quantum information processing and plays an important role in condensed matter physics. But, in fact, although the entanglement of multipartite systems can be similarly investigated as bipartite case, the properties and quantification of entanglement for higher dimensional systems and multipartite quantum systems are some issues still to be resolved.

On the other hand, as a combination of general relativity, quantum field theory and quantum information theory, the relativistic quantum information has been a focus of research in quantum information science over recent years for both conceptual and experimental reasons. In the last few years, much attention had been given to the study of entanglement shared between inertial and noninertial observers by discussing how the Unruh or Hawking effect will influence the degree of entanglement [4–19]. However, it is worth to note that most investigations in the noninertial system focused on the study of the quantum information in bipartite systems and only one of the subsystems accelerated. Fortunately, the tripartite entanglement of scalar field between noninertial frames was studied by Mi-Ra Hwang *et al.* [20] most recently. They showed that the tripartite entanglement decreases with increase of the acceleration but different from bipartite entanglement when one observer moves with an infinite acceleration.

In this paper we will discuss both the bipartite and tripartite entanglement of Dirac fields in the noninertial frame when one or two observers accelerated. We are interested in how the accelerations of these observers will influence the degree of bipartite and tripartite entanglement, and whether the differences between Fermi-Dirac and Bose-Einstein statistic will play a role in the entanglement decrease. Our setting consists of three observers: Alice, Bob and Charlie. We first assume Alice is in an inertial frame, Bob and Charlie are observing the system from accelerated frames, and then let Alice and Bob stay stationary while Charlie

moves with uniformly accelerations. We consider the Dirac fields, as shown in Refs. [21–23], from the perspective of the observers who uniformly accelerated, are described as an entangled state of two modes monochromatic with frequency $\omega_i, \forall i$

$$|0_{\omega_i}\rangle_M = \cos r_i |0_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II} + \sin r_i |1_{\omega_i}\rangle_I |1_{\omega_i}\rangle_{II}, \quad (1)$$

and the only excited state is

$$|1_{\omega_i}\rangle_M = |1_{\omega_i}\rangle_I |0_{\omega_i}\rangle_{II}, \quad (2)$$

where $\cos r_i = (e^{-2\pi\omega_i c/a_i} + 1)^{-1/2}$, a_i is the acceleration of the observer i . Considering that an accelerated observer in Rindler region I has no access to the field modes in the causally disconnected region II . By tracing over the inaccessible modes we will obtain a tripartite state and then we calculate the tripartite entanglement of the 3-qubit state as well as bipartite entanglement of all possible bipartite divisions of the tripartite system.

The outline of the paper is as follows. In Sec. II we recall some measurements of entanglement in quantum information theory, in particular the negativity and π -tangle. In Sec. III the bipartite and tripartite entanglement of Dirac fields when one or two of the observers accelerated will be discussed. The conclusions are presented in the last section.

II. MEASURES OF TRIPARTITE ENTANGLEMENT

It is well known that there are two remarkable entanglement measures for a bipartite system $\rho_{\alpha\beta}$, the concurrence [24] and the negativity [25]. The former is defined as

$$C_{\alpha\beta} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad \lambda_i \geq \lambda_{i+1} \geq 0, \quad (3)$$

where λ_i are the square roots of the eigenvalues of the matrix $\rho_{\alpha\beta} \tilde{\rho}_{\alpha\beta}$ with $\tilde{\rho}_{\alpha\beta} = (\sigma_y \otimes \sigma_y) \rho_{\alpha\beta}^* (\sigma_y \otimes \sigma_y)$ is the “spin-flip” matrix and σ_y is the Pauli matrix, and the latter is defined as

$$\mathcal{N}_{\alpha\beta} = \|\rho_{\alpha\beta}^{T_\alpha}\| - 1, \quad (4)$$

where T_α denotes the partial transpose of $\rho_{\alpha\beta}$ and $\|\cdot\|$ is the trace norm of a matrix. Corresponding, there are two entanglement measures which quantify the genuine tripartite entanglement: three-tangle [24] and π -tangle [26]. The three-tangle (or residual tangle), which

has many nice properties but it is a highly difficult problem to compute it analytically except few rare cases, is defined as

$$\tau_{\alpha,\beta,\gamma} = \tau_{\alpha(\beta,\gamma)} - \tau_{\alpha,\beta} - \tau_{\alpha,\gamma}, \quad (5)$$

where $\tau_{\alpha(\beta,\gamma)} = C_{\alpha(\beta,\gamma)}^2$ and $\tau_{\alpha,\beta} = C_{\alpha,\beta}^2$.

In order to get a easier calculation here we only adopt the π -tangle as the quantification of the tripartite entanglement. For any 3-qubit states $|\Phi\rangle_{\alpha\beta\gamma}$, the entanglement quantified by the negativity between α and β , between α and γ , and between α and the overall subsystem $\beta\gamma$ satisfies the following Coffman-Kundu-Wootters (CKW) monogamy inequality [24]

$$\mathcal{N}_{\alpha\beta}^2 + \mathcal{N}_{\alpha\gamma}^2 \leq \mathcal{N}_{\alpha(\beta\gamma)}^2, \quad (6)$$

where $\mathcal{N}_{\alpha\beta}$ is ‘two-tangle’, which is the negativity of the mixed state $\rho_{\alpha\beta} = \text{Tr}_\gamma(|\Phi\rangle_{\alpha\beta\gamma}\langle\Phi|)$ and $\mathcal{N}_{\alpha(\beta\gamma)}^2$ is ‘one-tangle’, defined as $\mathcal{N}_{\alpha(\beta\gamma)} = \|\rho_{\alpha(\beta\gamma)}^{T_\alpha}\| - 1$. The difference between the two sides of Eq.(6) can be interpreted as the residual entanglement

$$\pi_\alpha = \mathcal{N}_{\alpha(\beta\gamma)}^2 - \mathcal{N}_{\alpha\beta}^2 - \mathcal{N}_{\alpha\gamma}^2. \quad (7)$$

Likewise, we have

$$\pi_\beta = \mathcal{N}_{\beta(\alpha\gamma)}^2 - \mathcal{N}_{\beta\alpha}^2 - \mathcal{N}_{\beta\gamma}^2, \quad (8)$$

and

$$\pi_\gamma = \mathcal{N}_{\gamma(\alpha\beta)}^2 - \mathcal{N}_{\gamma\alpha}^2 - \mathcal{N}_{\gamma\beta}^2. \quad (9)$$

The π -tangle $\pi_{\alpha\beta\gamma}$ is defined as the average of π_α , π_β , and π_γ , i.e.,

$$\pi_{\alpha\beta\gamma} = \frac{1}{3}(\pi_\alpha + \pi_\beta + \pi_\gamma). \quad (10)$$

III. BEHAVIORS OF TRIPARTITE ENTANGLEMENT WHEN ONE OR TWO OBSERVER ACCELERATED

We consider a tripartite system which is consist of three subsystems, name Alice as the observer of the first part of the system, Bob and Charlie are the observers of the second and third parts respectively. They shared a GHZ state

$$|\Phi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C), \quad (11)$$

where $|0\rangle_{A(B,C)}$ $|1\rangle_{A(B,C)}$ are vacuum states and from an inertial observer. Using Eqs. (4) and (10), we can easily get

$$\begin{aligned}\mathcal{N}_{A(BC)} &= \mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)} = 1, \\ \mathcal{N}_{AB} &= \mathcal{N}_{BC} = \mathcal{N}_{CA} = 0, \\ \pi_{ABC} &= 1,\end{aligned}$$

where $\mathcal{N}_{A(BC)}$, \mathcal{N}_{AB} and π_{ABC} are the ‘one-tangle’, ‘two-tangle’ and ‘ π -tangle’ of state (11) from a inertial viewpoint. Then we let Alice stays stationary while Bob and Charlie move with uniform accelerations. Since Bob and Charlie are accelerated, we should map the second and third partition of this state into the Rindler Fock space basis. Using Eqs. (1) and (2) we can rewrite Eq. (11) in terms of Minkowski modes for Alice and Rindler modes for Bob and Charlie

$$\begin{aligned}|\Phi\rangle_{AB_IC_I} &= \frac{1}{\sqrt{2}} \left[\cos r_b \cos r_c |0\rangle_A |0\rangle_{B_I} |0\rangle_{B_{II}} |0\rangle_{C_I} |0\rangle_{C_{II}} + \cos r_b \sin r_c |0\rangle_A |0\rangle_{B_I} |0\rangle_{B_{II}} |1\rangle_{C_I} |1\rangle_{C_{II}} \right. \\ &\quad + \cos r_b \sin r_c |0\rangle_A |1\rangle_{B_I} |1\rangle_{B_{II}} |0\rangle_{C_I} |0\rangle_{C_{II}} + \sin r_b \sin r_c |0\rangle_A |1\rangle_{B_I} |1\rangle_{B_{II}} |1\rangle_{C_I} |1\rangle_{C_{II}} \\ &\quad \left. + |1\rangle_A |1\rangle_{B_I} |0\rangle_{B_{II}} |1\rangle_{C_I} |0\rangle_{C_{II}} \right].\end{aligned}\quad (12)$$

Let us first calculate the one-tangle between subsystem A and the overall subsystem B_IC_I by use of Eq. (4). Tracing over the inaccessible modes B_{II} and C_{II} we obtain a density matrix

$$\begin{aligned}\rho_{AB_IC_I} &= \frac{1}{2} \left[\cos^2 r_b \cos^2 r_c |000\rangle\langle 000| + \cos^2 r_b \sin^2 r_c |001\rangle\langle 001| \right. \\ &\quad + \sin^2 r_b \cos^2 r_c |010\rangle\langle 010| + \sin^2 r_b \sin^2 r_c |011\rangle\langle 011| \\ &\quad \left. + \cos r_b \cos r_c (|111\rangle\langle 000| + |000\rangle\langle 111| + |111\rangle\langle 111|) \right],\end{aligned}\quad (13)$$

where $|lmn\rangle = |l\rangle_A |m\rangle_{B_I} |n\rangle_{C_I}$. Then we can easily get the partial transpose subsystem A of Eq. (13)

$$\begin{aligned}\rho_{AB_IC_I}^{T_A} &= \frac{1}{2} \left[\cos^2 r_b \cos^2 r_c |000\rangle\langle 000| + \cos^2 r_b \sin^2 r_c |001\rangle\langle 001| \right. \\ &\quad + \sin^2 r_b \cos^2 r_c |010\rangle\langle 010| + \sin^2 r_b \sin^2 r_c |011\rangle\langle 011| \\ &\quad \left. + \cos r_b \cos r_c (|011\rangle\langle 100| + |100\rangle\langle 011| + |111\rangle\langle 111|) \right],\end{aligned}\quad (14)$$

from which we can get $(\rho_{AB_IC_I}^{T_A})^\dagger$ and the negativity $\mathcal{N}_{A(B_IC_I)}$ is found to be

$$\mathcal{N}_{A(B_IC_I)} = \frac{1}{2} \left[\cos r_b \cos r_c + \cos^2 r_c + \cos^2 r_b \sin^2 r_c + \sqrt{\cos^2 r_b \cos^2 r_c + \sin^4 r_b \sin^4 r_c - 1} \right] \quad (15)$$

Similarly, we can also get

$$\mathcal{N}_{B_I(AC_I)} = \frac{1}{2} \left[\cos r_b \cos r_c + \cos^2 r_b + \sin^2 r_b \sin^2 r_c + \cos r_c \sqrt{\cos^2 r_b + \sin^4 r_b \cos^2 r_c - 1} \right], \quad (16)$$

and

$$\mathcal{N}_{C_I(AB_I)} = \frac{1}{2} \left[\cos r_b \cos r_c + \sin^2 r_b + \cos^2 r_b \cos^2 r_c + \cos r_b \sqrt{\cos^2 r_c + \sin^4 r_c \cos^2 r_b - 1} \right]. \quad (17)$$

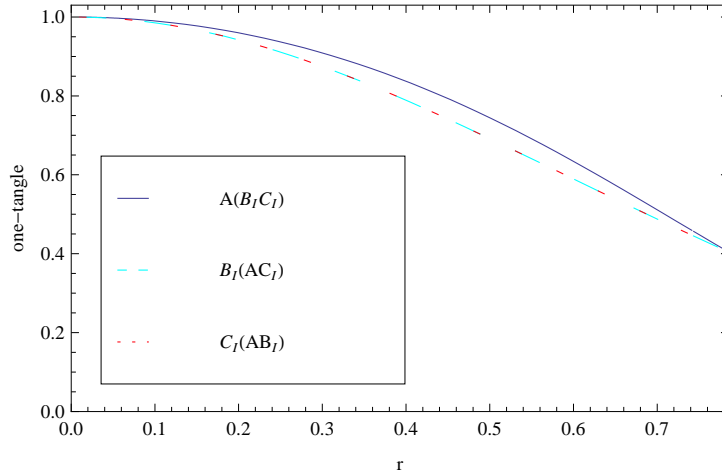


FIG. 1: (Color online) The negativity $\mathcal{N}_{A(B_IC_I)}$ (solid line), $\mathcal{N}_{B_I(AC_I)}$ (dashed line), and $\mathcal{N}_{C_I(AB_I)}$ (dotted line) of two-observers-accelerated case as a function of the acceleration parameter $r = r_b = r_c$.

The properties of all the one-tangles of $\rho_{AB_IC_I}$ are shown in Fig. 1 with $r_b = r_c = r$. It is shown that all the one-tangles equal to one when $r = 0$, which is exactly the value of one-tangles in Eq. (11) obtained in the inertial frame. All of them decrease as the accelerations of Bob and Charlie increase, which is similar to the behaviors of bipartite entanglement of

Dirac field [6] and tripartite one-tangle of scalar field when one of the observers accelerated [20]. Note that $\mathcal{N}_{B_I(AC_I)} = \mathcal{N}_{C_I(AB_I)}$ for all accelerations which indicate Bob and Charlie's subsystems is symmetry in this case. It is worthwhile to note that unlike the scalar case the one-tangle $N_{C_I(AB)}$ goes to zero when Charlie moves with infinite acceleration, here $N_{C_I(AB_I)}$ and $N_{C_I(AB)}$ never goes to zero for any acceleration. We argue that this different is due to the differences between Fermi-Dirac and Bose-Einstein statistics [10] rather than because the observers cannot access to the entanglement of the subsystems who moves with infinite acceleration respect to them, as the authors stated in Ref. [20]. What's surprising is that in the case of both Bob and Charlie move with infinite accelerations ($r = \pi/4$), $\mathcal{N}_{A(B_IC_I)} = \mathcal{N}_{B_I(AC_I)} = \mathcal{N}_{C_I(AB_I)} = \frac{1-\sqrt{5}}{8}$, which means that there is no difference between all the subsystems A , B_I and C_I in this limit.

Now, let us compute the two-tangle between subsystems A and B_I , tracing the qubit of subsystem C_I we obtain

$$\rho_{AB_I} = \frac{1}{2} \begin{pmatrix} \cos r_b^2 \cos^2 r_c + \cos r_b^2 \sin^2 r_c & 0 & 0 & 0 \\ 0 & \sin^2 r_b \cos^2 r_c + \sin r_b^2 \sin^2 r_c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Using this matrix and Eq.(4) we can obtain the negativity $\mathcal{N}_{AB_I} = 0$, which means there is no bipartite entanglement between mode A and B_I in spite of Bob and Charlie with accelerations. Similarly, it is found that $\mathcal{N}_{AC_I} = \mathcal{N}_{B_IC_I} = 0$. Note that the CKW inequality [24], $\mathcal{N}_{\alpha\beta}^2 + \mathcal{N}_{\alpha\gamma}^2 \leq \mathcal{N}_{\alpha(\beta\gamma)}^2$, is saturated for any acceleration parameter r .

Then by use of Eqs. (7~10), the π -tangle of our system is found to be

$$\pi_{AB_IC_I} = \frac{1}{3}(\pi_A + \pi_{B_I} + \pi_{C_I}) = \frac{1}{3} \left[\mathcal{N}_{A(B_IC_I)}^2 + \mathcal{N}_{B_I(AC_I)}^2 + \mathcal{N}_{C_I(AB_I)}^2 \right], \quad (18)$$

where $\mathcal{N}_{A(B_IC_I)}$, $\mathcal{N}_{B_I(AC_I)}$ and $\mathcal{N}_{C_I(AB_I)}$ are given by Eq.(15~17) respectively.

In order to better understand the multipartite entanglement in the noninertial frames, we also compute the entanglement of a tripartite system include two inertial subsystems and one noninertial subsystem, i.e., let Alice and Bob stay stationary and Charlie moves with uniform acceleration. They shared the same GHZ state Eq. (11) at the same point in the

Minkowski spacetime. According to the preceding calculations, we can obtain

$$\begin{aligned}
\mathcal{N}_{A(BC_I)} &= \mathcal{N}_{B(AC_I)} = \cos r_c, \\
\mathcal{N}_{C_I(AB)} &= \frac{1}{2}(\cos r_c + \cos^2 r_c + \sqrt{\cos^2 r_c + \sin^4 r_c} - 1), \\
\mathcal{N}_{AB} &= \mathcal{N}_{BC_I} = \mathcal{N}_{C_I A} = 0,
\end{aligned} \tag{19}$$

where $\mathcal{N}_{A(BC_I)}$ and \mathcal{N}_{AB} are the one-tangle and two-tangle of the one-observer-accelerated case. It is worthy to notice that the CKW inequality is also saturated for any acceleration parameter r in this case. From these facts we arrive at the conclusion that this inequality is valid in both inertial and noninertial frames.

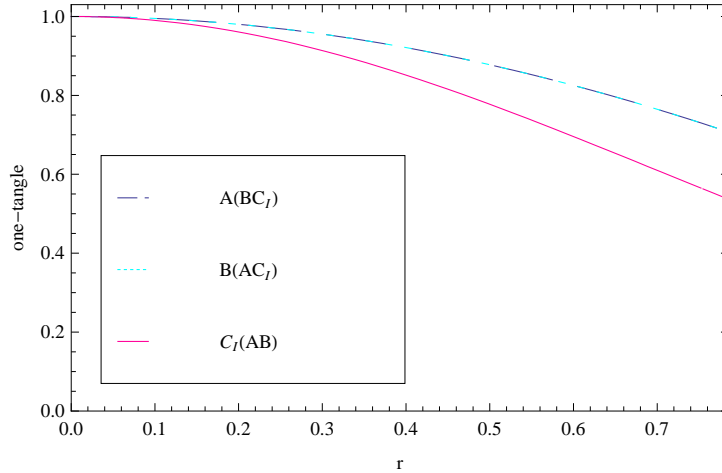


FIG. 2: (Color online) The one-tangles $\mathcal{N}_{A(BC_I)}$ (dashed line), $\mathcal{N}_{B(AC_I)}$ (dotted line), and $\mathcal{N}_{C_I(AB)}$ (solid line) of one-observer-accelerated case as a function of the acceleration parameter $r = r_c$.

We plot the one-tangles of this case in Fig. 2 and find that: (i) all of them decrease as the acceleration of Charlie increases; (ii) $\mathcal{N}_{A(BC_I)} = \mathcal{N}_{B(AC_I)}$ for all accelerations; (iii) the one-tangle $\mathcal{N}_{C_I(AB)}$ never goes to zero for any acceleration. However, it is interesting to note that in this case $\mathcal{N}_{A(BC_I)} = \mathcal{N}_{B(AC_I)} \neq \mathcal{N}_{C_I(AB)}$ when Charlie moves with infinite acceleration, which is very different from the two-observers-accelerated case. We are not sure whether this is an individual case only appears in the fermionic systems because it probably related to the incomplete definition of the one-tangle in the noninertial frames. Thus, it seems to be interesting to repeat the calculation of this paper for other systems and by making use of other entanglement measurements. It is shown again that all the two-tangles equal to zero in this case, which is exactly the same as the two-tangles obtained in the inertial frame. That

is to say, either one or two subsystems of the tripartite state are accelerated, there is no bipartite entanglement in this system. The acceleration doesn't generate bipartite entanglement and the entanglement structure of the quantum state doesn't change. It is interesting to note that in Ref. [6], there was no tripartite entanglement between observers Alice, Rob and Anti-Rob, i.e., all the entanglement is bipartite when one of the observers is static and the other accelerated. However, here we find that there is no bipartite entanglement, all the entanglement of this system is in form of tripartite entanglement.

By use of Eq. (12) we get the π -angle of the one-observer-accelerated system $\pi_{ABC_I} = \frac{\mathcal{N}_{A(BC_I)}^2 + \mathcal{N}_{B(AC_I)}^2 + \mathcal{N}_{C_I(AB)}^2}{3}$. For comparison, we plot the π_{ABC_I} of this case and the $\pi_{AB_IC_I}$ of two-observers-accelerated case in Fig. 3.

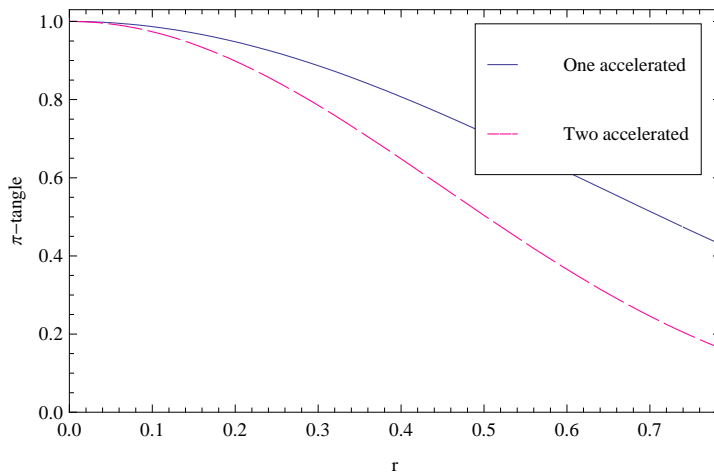


FIG. 3: (Color online) The π -angle π_{ABC_I} of one-observer-accelerated case (solid line) versus $\pi_{AB_IC_I}$ of two-observers-accelerated case (dashed line), as a function of the acceleration parameter $r = r_b = r_c$.

In Fig. (3) we plot the π -angle π_{ABC_I} of the one-observer-accelerated case and $\pi_{AB_IC_I}$ of the two-observers-accelerated case as a function of the acceleration parameter $r = r_b = r_c$, which shows how the acceleration changes tripartite entanglement. In the case of zero acceleration, $\pi_{ABC_I} = \pi_{AB_IC_I} = 1$. With the increasing of accelerations, the π -angles decrease monotonously for both of these two cases. It is shown that the decrease speed of the tripartite entanglement of two-observers-accelerated case is much quicker than that of one-observer-accelerated case as expected. Recall that the loss of entanglement was explained as the information formed in the inertial system was leaked into the causally disconnected region

[14, 19, 22] due to the Unruh effect. The quicker decreases of entanglement in two-observer-accelerated case is attribute to the information both in Bob and Charlie's subsystems are redistributed into the inaccessible regions as the growth of acceleration. In the limit of infinite acceleration, the tripartite entanglement of two-observers-accelerated case reduces to a lower minimum but never vanishes. It is interesting to note that either the scalar system or Dirac system and either one or two observers are accelerated, the tripartite entanglement never vanishes for any acceleration. Thus we can arrive a striking conclusion that the quantum entanglement in tripartite system is a better resource to perform quantum information processing such as teleportation. We can also perform such quantum information tasks by use of the tripartite entanglement when some observers are falling into a black hole while others are hovering outside the event horizon.

IV. SUMMARY

The effect of acceleration on bipartite and tripartite entanglement of a 3-qubit Dirac system when one or two subsystems accelerated is investigated. It is shown that all the one-tangles decrease as the accelerations of Bob and Charlie increase. However, unlike the scalar case in which the one-tangle $\mathcal{N}_{C_I(AB)}$ goes to zero when Charlie moves with infinite acceleration, here $\mathcal{N}_{C_I(AB_I)}$ and $\mathcal{N}_{C_I(AB)}$ never reduce to zero for any acceleration. It is also shown that the CKW inequality is valid in the noninertial systems. It is of interest to note that $\mathcal{N}_{A(B_I C_I)} = \mathcal{N}_{B_I(A C_I)} = \mathcal{N}_{C_I(A B_I)}$ in the infinite acceleration limit, which means that there is no difference between all the subsystem A , B_I and C_I in this limit. It is found that either one or two subsystems of the tripartite state accelerated, there is no bipartite entanglement in this system, i.e., all entanglement of this system is in form of tripartite entanglement. The acceleration doesn't generate bipartite entanglement in this system and doesn't change the entanglement structure of the quantum state. It is also found that the π -tangle of the two-observers-accelerated case decreases much quicker than that of the one-observer-accelerated case and it reduces to a non-zero minimum in the infinite acceleration limit. It is worthy to mention that both for scalar [20] and Dirac fields, and either one or two observers are accelerated, the tripartite entanglement doesn't vanish for any acceleration. That is to say, the quantum entanglement in tripartite system is a better resource than bipartite entanglement to perform quantum information processing such as teleportation.

We can also perform such quantum information tasks by use of tripartite entanglement when one or two observers are falling into a black hole while others hovers outside the event horizon. The discussions of this paper can be also applied to the investigations of multipartite entanglement and quantum correlations in the curved spacetime [17, 18, 22] as well as the properties of multipartite Gaussian entanglement in noninertial frames [14]. Therefore, further investigation by using the results in this paper will not only help us deeply understand genuine multipartite entanglement but also helpful to give a better insight into the entanglement entropy and information paradox of black holes.

Acknowledgments

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- [1] A. Peres and D. R. Terno, *Rev. Mod. Phys.* **76**, 93 (2004).
 - [2] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, *Phys. Rev. Lett.* **80**, 1121 (1998).
 - [3] D. Bouwmeester, A. Ekert, and A. Zeilinger, *The Physics of Quantum Information* (Springer-Verlag, Berlin), 2000.
 - [4] P. M. Alsing and G. J. Milburn, *Phys. Rev. Lett.* **91**, 180404 (2003).
 - [5] I. Fuentes-Schuller and R. B. Mann, *Phys. Rev. Lett.* **95**, 120404 (2005).
 - [6] P. M. Alsing, I. Fuentes-Schuller, R. B. Mann, and T. E. Tessier, *Phys. Rev. A* **74**, 032326 (2006).
 - [7] T. C. Ralph, G. J. Milburn, and T. Downes, *Phys. Rev. A* **79**, 022121 (2009).
 - [8] J. Doukas and L. C. L. Hollenberg, *Phys. Rev. A* **79**, 052109 (2009).
 - [9] S. Moradi, *Phys. Rev. A* **79**, 064301 (2009).
 - [10] E. Martn-Martnez and J. Len, *Phys. Rev. A* **80**, 042318 (2009); *Phys. Rev. A* **81**, 032320 (2010).

- (2010); Phys. Rev. A **81**, 052305 (2010).
- [11] J. Wang, J. Deng and J. Jing, Phys. Rev. A **81**, 052120 (2010); J. Wang and J. Jing, Phys. Rev. A **82**, 032324 (2010).
 - [12] David C. M. Ostapchuk and R. B. Mann, Phys. Rev. A **79**, 042333 (2009).
 - [13] Andr G. S. Landulfo and George E. A. Matsas, Phys. Rev. A **80**, 032315 (2009).
 - [14] G. Adesso, I. Fuentes-Schuller, and M. Ericsson, Phys. Rev. A **76**, 062112 (2007).
 - [15] J. Len and E. Martn-Martnez, Phys. Rev. A **80**, 012314 (2009).
 - [16] R. B. Mann and V. M. Villalba, Phys. Rev. A **80**, 022305 (2009).
 - [17] Q. Pan and J. Jing, Phys. Rev. A **77**, 024302 (2008); Phys. Rev. D **78**, 065015 (2008).
 - [18] J. Wang, Q. Pan, and J. Jing, Ann. Phys. **325**, 1190 (2010); Phys. Lett. B **692**, 202 (2010).
 - [19] J. Wang, Q. Pan, S. Chen, and J. Jing, Phys. Lett. B **677**, 186 (2009); Quant. Inf. Comput. **10**, 0947 (2010).
 - [20] M.-R. Hwang, D. Park, and E. Jung, arXiv:1010.6154v1.
 - [21] M. Aspachs, G. Adesso, and I. Fuentes, Phys. Rev. Lett **105** , 151301 (2010).
 - [22] E. Martín-Martínez, L. J. Garay and Juan León, Phys. Rev. D **82**, 064006 (2010); Phys. Rev. D **82**, 064028 (2010).
 - [23] D. E. Bruschi, J. Louko, E. Martn-Martnez, A. Dragan, and I. Fuentes, Phys. Rev. A **82**, 042332 (2010).
 - [24] V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).
 - [25] G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002); M. B. Plenio, Phys. Rev. Lett. **95**, 090503 (2005).
 - [26] Y. U. Ou and H. Fan, Phys. Rev. A **75**, 062308 (2007).